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GROWTH RATE OF A VAPOR BUBBLE IN AN UNDERHEATED LIQUID

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On the basis of model representations developed earlier the author has obtained an analytical solution of the problem of vapor bubble growth on a wall in an underheated liquid.

A theoretical solution of the problem of growth of a vapor bubble in an unbounded volume of a uniformly superheated liquid has been obtained by Plesset and Zwick [1]:

$$\frac{R}{\sqrt{at}} = 2\mathrm{Ja}\,. \tag{1}$$

The growth rate of vapor bubbles on a solid wall is described well by the relation [2]

$$\frac{R}{\sqrt{at}} = \gamma\,\mathrm{Ja} + [(\gamma\,\mathrm{Ja})^2 + 2\beta\,\mathrm{Ja}]^{1/2}\,, \tag{2}$$

where $\gamma = 0.3$ and $\beta = 6$ are empirical constants. At $\mathrm{Ja} << 1$ expression (2) transforms into the well-known Labuntsov formula [3, 4] for the region of high pressures

$$\frac{R}{\sqrt{at}} = (2\beta\,\mathrm{Ja})^{1/2}\,. \tag{3}$$

It should be noted that in the cases of volume boiling up and surface boiling the time of bubble growth accessible to observation does not exceed 0.1 ... 0.15 sec [5]. In this respect, the experimental data obtained in [6] under conditions of weightlessness are unique in recording growth times up to 10 sec.

As is seen in Fig. 1, with boiling of a saturated liquid the results of [6] are described well by relation (1). For liquid underheating up to the saturation temperature the experimental dependences of the bubble radius on the time are described in [6] by the relation

$$R = At^{1/3}\,, \tag{4}$$

where A decreases with increase in underheating. Below we suggest an approximate model of bubble growth on a solid surface in boiling of an underheated liquid.

We write the thermal balance for a spherical bubble

$$Q_+ - Q_- = 4\pi r \rho'' R^2 \frac{dR}{dt}\,. \tag{5}$$

The heat transferred from the superheated liquid to the lower part of a bubble is determined by the relation [1]

$$Q_+ = 4\pi \frac{\lambda\,\Delta T_+ R^2}{\sqrt{at}}\,. \tag{6}$$

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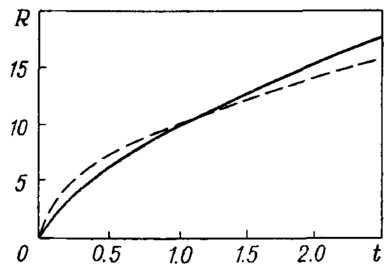

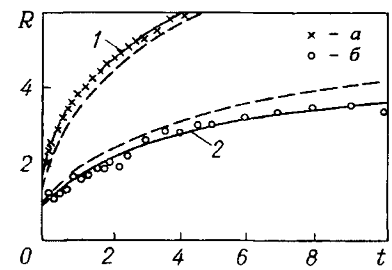

Fig. 1. Comparison of experimental data [6] on the growth rate of vapor bubbles in the boiling of saturated coolant R-113 at atmospheric pressure (the solid curve, obtained by averaging more than 50 experimental points) with the theoretical solution [1] (the dashed curve). R, mm; t, sec.

Fig. 2. Comparison of experimental data [6] on the growth rate of vapor bubbles under boiling conditions for underheated coolant R-113 (points) at atmospheric pressure with the exact solution (11) and (12) (the solid curve) and the approximate solution (14) (the dashed curve): a, 1) $\Delta T_- =$ 5 K; b, 2) 25.

To determine the heat released by the upper part of a bubble of the cold liquid due to the condensation process, we will use the model of [7] for turbulent transfer in a liquid in the vicinity of a growing bubble. Now we introduce the coefficient of turbulent transfer of momentum and heat

$$\varepsilon \approx l^2 \omega . \tag{7}$$

We will proceed from the assumption that in the period of bubble growth, pulsations with an amplitude proportional to its radius $l \approx R$ develop on its surface. We assume that the frequency of the oscillations is equal to the natural frequency of bubble pulsations determined by the Rayleigh formula $\omega \approx \sqrt{\sigma/\rho R^3}$. Then for the heat released from the upper part of a bubble of the underheated liquid we can write an expression in the form of (6) if we substitute in the latter the turbulent thermal conductivity $\lambda_* = \rho c_p \varepsilon$ for the molecular thermal conductivity λ and the liquid underheating ΔT_- for the liquid superheating ΔT_+:

$$Q_- = k c_p \Delta T_- R^2 \sqrt{\left(\frac{\rho \sigma R}{at}\right)} . \tag{8}$$

Here k is an empirical constant, and from physical considerations we will have $k << 1$ (the amplitude of the bubble surface oscillations is much smaller than its radius; condensation occurs only on part of the bubble surface).

Substitution of (6)-(8) into (5) yields

$$\frac{dR}{dt} = \frac{\lambda \Delta T_+}{r \rho'' \sqrt{at}} \left(1 - k \frac{\Delta T_-}{\Delta T_+} \frac{1}{a} \sqrt{\left(\frac{\sigma R}{\rho}\right)}\right) . \tag{9}$$

From (9) it follows that as $t \to \infty$ the bubble radius tends to its maximum value

$$R_{\max} = \frac{1}{k^2} \frac{\rho a^2}{\sigma} \left(\frac{\Delta T_+}{\Delta T_-}\right)^2 . \tag{10}$$

Integration of (9) with the initial condition $t = 0 : R = 0$ leads to an analytical solution of the problem of vapor bubble growth in an underheated liquid in the implicit form

$$y = x - \ln x - 1 \, . \tag{11}$$

Here we have used the following notation:

$$x = 1 - z \, ; \quad z = k \frac{\Delta T_-}{\Delta T_+} \frac{1}{a} \sqrt{\left(\frac{\sigma R}{\rho} \right)} ; \quad y = k^2 \frac{\sigma}{\rho a^2} \left(\frac{\Delta T_-}{\Delta T_+} \right)^2 \mathrm{Ja} \sqrt{at} \, . \tag{12}$$

As Fig. 2 shows, at $k \approx 2 \cdot 10^{-4}$ solution (11) and (12) agrees well with the experimental data of [6].

At $z << 1$, from (11), (12) the asymptotic solution

$$y \approx \frac{z^2}{2} + \frac{z^3}{3} + \frac{z^4}{4} + \ldots \frac{z^n}{n} + \ldots \tag{13}$$

follows. Retaining the first term $y \approx z^2/2$ in the right-hand side of (13), we arrive at the theoretical solution (1) for conditions of a saturated liquid. If we now assume that approximately $y \approx z^3/3$, we obtain an expression of the form of (4):

$$R \approx k_1 \left(\frac{3}{k} \right)^{2/3} \left(\frac{c_p \rho^2 \Delta T_+^2 a^3 t}{r \rho'' \sigma \Delta T_-} \right)^{1/3} . \tag{14}$$

Here, the new empirical constant, $k_1 \approx 3 \cdot 10^2$ is introduced. As is seen from Fig. 2, the approximate solution (14) agrees with the experimental data of [6] slightly worse than the exact solution (11) and (12); however, as a whole it is quite satisfactory.

Relation (14) can be used to extend the "model of boiling turbulent heat transfer" developed in [7] to the case of final liquid underheating.

NOTATION

t, time; ρ, c_p, λ, and a, density, specific heat, thermal conductivity, and thermal diffusivity of the liquid, respectively; ρ'', density of the saturated vapor; r, heat of the phase transition; σ, coefficient of surface tension; ΔT_+ and ΔT_-, superheating and underheating relative to the saturation temperature; Q_+, heat transfered from the superheated liquid to the bubble bottom; Q_-, heat released from the upper part of a bubble in the underheated liquid; ε, coefficient of turbulent transfer of momentum and heat in the liquid; λ_*, coefficient of turbulent thermal conductivity; R, bubble radius; l, pulsation amplitude of the bubble surface; ω, pulsation frequency of the bubble surface; $\mathrm{Ja} = \rho c_p \Delta T_+ / r \rho''$, Jacobi number.

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